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Book Review: *Aspects and Applications of The Random Walk*

Aspects and Applications of the Random Walk. G. H. Weiss, North-Holland, New York, 1994.

Scientists have often pondered the "unreasonable effectiveness" of mathematics in describing the complex cooperative phenomena which occur at many scales in our physical environment—processes ranging from subatomic to cosmic dimensions. A striking aspect of these mathematical models is the "universality" of the governing equations such that the same mathematical framework can encompass an accurate description of diverse types of material properties and material systems. This "universality" leads to many "analogies" which often make statistical physics research especially interesting. The success of random walk models, and the limit theorems associated with these walks, in describing the origin of various kinds of cooperative phenomena (phase transitions, fluid flow,...) is similarly remarkable and we can expect the perspective and tools of random walk theory to offer clues into the microscopic origin of large-scale regularities in the natural world and into the success of hydrodynamic mathematical models introduced to describe dynamic and equilibrium cooperative phenomena. George Weiss offers an initiation into this wonderful area of mathematical and physical investigation.

The wide applicability of random walk models and the interesting mathematics of random walk theory, however, have led to a large literature on random walks which is widely dispersed among technical and mathematical journals. Consequently, it is difficult for a student to obtain a basic grounding in the mathematical methods of random walk theory and an overview of the many applications of this theory. Good summaries of random walk theory exist which are tailored to the needs and interests of mathematicians, but there are no similar authoritative texts on random walk theory which emphasize the slant of a physical scientist. Weiss' text is targeted to reduce this gap. His book focuses on an introductory level presentation intended for the beginning graduate student with a physical

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science orientation and for advanced researchers in various applied disciplines who have become interested in random walk applications in their own field of interest.

Chapters 1–3 establish the basic mathematical tools of (discrete) random walk theory which frequently arise in applications. Given the introductory nature of the text, the discussion appropriately begins at an elementary level, where basic probability concepts are defined, but the development rapidly increases in sophistication. An elegant exposition of the Pearson random walk model is given which illustrates the formal probabilistic introduction. Lévy random walks and continuous-time random walks are also introduced at an early stage (Chapter 2) along with some very powerful computation tools for extracting asymptotic properties of random walks (the saddle-point approximation, Abelian and Tauberian theorems, etc.). A clear discussion of the mathematical mechanism underlying the central limit theorem (and its generalization to random processes for random variable without a finite variance) is given in Chapter 3 along with a physical discussion of the approximation of discrete random walks by Brownian motion in the continuum limit. Some subtleties of the continuum limit are appropriately emphasized.

The theoretical framework established in Chapters 1–3 is first applied to the classic problem of random walks on lattices in Chapter 4. Basic random walk concepts such as "transience," "recurrence," "number of distinct sites visited," "average recurrence time," "number of returns to origin," and "occupancy of set by a random walk" are introduced and economical derivations of many asymptotic random walk properties are summarized. This chapter illustrates the power of the "generating function" approach to random walk theory pioneered by Feller in his "fluctuation theory of recurrent events" and refined by Montroll and Weiss, and others thereafter, in the specific context of discrete random walk theory. Many of the classical asymptotic properties of random walks are summarized in Chapter 4 and it was enjoyable to read such a systematic and simple exposition of these results.

More specialized techniques are introduced in Chapters 5 and 6 to model random walks subject to constraints. The discussion begins with the presence of plane boundaries and the appropriate modeling of the boundary condition to model diffusion-limited reaction phenomena. A good discussion of the Dirichlet ("absorbing"), Neumann ("reflecting"), and Robin ("partially absorbing") boundary conditions in continuum diffusion equations is given for the corresponding discrete random walk analogs. The introduction of additional boundaries naturally leads to a discussion of "first-passage times," and "spans" of random walks, and Wald's identity for estimating the asymptotic properties of constrained random walks. The last part of Chapter 5 focuses on the "trapping problem" in which the simple plane surface constraint is replaced by closed surfaces placed at random which obstruct the motion of particles undergoing random walks and which can react with the random walking particles. This topic touches on many issues of current scientific interest and brings this introductory text to a sophisticated level.

Chapter 6 specializes the discussion to a variety of multistate random walk models, which are natural extensions of the continuous-time random walk model introduced in earlier chapters. The applications of these models to describe chromatographic processes provide the highlight of this chapter.

Chapter 7, the final chapter, describes applications and illustrates many of the previous formal developments. Some of the applications considered are "standards" such as the Scher–Montroll model of "anomalous" electrical transport in amorphous semiconductors, which provides a good application of the continuous-time random walk model, and random walks on "combs" are considered as toy models of diffusion on tenuously connected structures such as percolation clusters. The applications also include less familiar discussions of the inference of structural crystallographic information from x-ray scattering data and the interpretation of the scattering of light from human tissue based on random walk modeling. These latter applications provide examples in which the random walk modeling is both natural and practically important and these sections should especially appeal to novice random walk modelers.

Weiss' text is clearly written and provides a good overview of mathematical techniques which frequently arise in discrete random walk modeling. A minor weakness of the text is that the applications considered are limited to a narrow range and these applications are isolated from the theoretical development, so that the text has perhaps too much of a mathematical flavor for many physical scientists. There also could have been further development of the asymptotics of continuum random walk models based on Wiener path-integration. There are many applications associated with continuum (Brownian) motion subject to constraints naturally added by "weighing" the paths in the path-integral representation of random walk properties. A brief mention of these important techniques is given in Chapter 4 in a discussion of the "occupancy time of a set" by Brownian paths. These mathematical methods, which are central to many field-theoretic and polymer physics applications, deserve further attention even in an introductory text.

This book will be useful to researchers who would like to familiarize themselves with the basic results and techniques of discrete random walk theory. Weiss' clear discussion often sheds new light on old problems for more experienced readers and he provides many useful references to recent applications of random walk theory. The style of the book makes it suitable for supplemental reading in a variety of specialized courses in which random walk applications arise and it should also serve as a valuable desk reference for more experienced statistical physics and materials science researchers.

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